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The algebraic and analytic compactifications of the Hitchin moduli space

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Abstract

Following the work of Mazzeo, Swoboda, Weiß and Witt (Duke Math. J. **165** (2016), no. 12, 2227–2271) and of Mochizuki (J. Topol. **9** (2016), no. 4, 1021–1073), there is a map $\overline{\Xi}$ between the algebraic compactification of the Dolbeault moduli space of $\mathrm{SL}(2,\mathbb{C})$ Higgs bundles on a smooth projective curve coming from the $\mathbb{C}^*$ action, and the analytic compactification of Hitchin's moduli space of solutions to the SU(2) self-duality equations on a Riemann surface obtained by adding solutions to the decoupled equations, known as 'limiting configurations'. This map extends the classical Kobayashi–Hitchin correspondence. The main result of this paper is that $\overline{\Xi}$ fails to be continuous at the boundary over a certain subset of the discriminant locus of the Hitchin fibration.

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